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# The Heisenberg antiferromagnet on an anisotropic triangular lattice: linear spin-wave theory

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Received 6 January 1999

**Abstract.** We consider the effect of quantum spin fluctuations on the ground-state properties of the Heisenberg antiferromagnet on an anisotropic triangular lattice using linear spin-wave (LSW) theory. This model should describe the magnetic properties of the insulating phase of the  $\kappa$ -(BEDT-TTF)<sub>2</sub>X family of superconducting molecular crystals. The ground-state energy, the staggered magnetization, magnon excitation spectra, and spin-wave velocities are computed as functions of the ratio of the antiferromagnetic exchange between the second and first neighbours,  $J_2/J_1$ . We find that near  $J_2/J_1 = 0.5$ , i.e., in the region where the classical spin configuration changes from a Néel-ordered phase to a spiral phase, the staggered magnetization vanishes, suggesting the possibility of a quantum disordered state. In this region, the quantum correction to the magnetization is large but finite. This is in contrast to the case for the frustrated Heisenberg model on a square lattice, for which the quantum correction diverges logarithmically at the transition from the Néel to the collinear phase. For large  $J_2/J_1$ , the model becomes a set of chains with frustrated interchain coupling. For  $J_2 > 4J_1$ , the quantum correction to the magnetization, within LSW theory, becomes comparable to the classical magnetization, suggesting the possibility of a quantum disordered state. We show that, in this regime, the quantum fluctuations are much larger than for a set of weakly coupled chains with non-frustrated interchain coupling.

## 1. Introduction

The study of strongly correlated electron systems in low dimensions is a very active field of research. One of the great challenges is to understand the competition between antiferromagnetism and superconductivity found in cuprate and organic superconductors. Kino and Fukuyama [1] recently proposed interacting electron models for a range of BEDT-TTF crystals. McKenzie argued that the  $\kappa$ -(BEDT-TTF)<sub>2</sub>X family can be described by a simplified version of one of their models, a Hubbard model on an anisotropic triangular lattice [2]. Recent quantum Monte Carlo calculations [3] and calculations at the level of the random-phase approximation [4] and the fluctuation-exchange approximation [5] suggest that at the boundary of the antiferromagnetic phase, this model exhibits superconductivity mediated by spin fluctuations. As the anisotropy of the intersite hopping varies, the model changes from the square lattice to the isotropic triangular lattice to decoupled chains [2]. The wavevector associated with the antiferromagnetic spin fluctuations changes and the superconductivity has been predicted to change from d-wave singlet (as in the cuprates) to s-wave triplet in the odd-frequency channel for the isotropic triangular lattice [4]. This shows that an understanding of the antiferromagnetic interactions is important for understanding the symmetry of the

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Cooper pairs in the superconducting state. As suggested by Seo and Fukuyama [6],  $\theta$ -(BEDT-TTF)<sub>2</sub>RBZn(SCN)<sub>4</sub> can be described by the model that we consider with  $J_2/J_1 \approx 5$ . Experimental findings from Mori *et al* [7] show that this material has a spin gap. The same model has also been proposed by Horsch and Mack [8] to be relevant to  $\alpha'$ -NaVO<sub>5</sub>.

The Heisenberg model studied here should also describe the magnetic properties of the molecular crystals  $\kappa$ -(BEDT-TTF)<sub>2</sub>X with X = Cu[N(CN)<sub>2</sub>]Cl, Cu(CN)<sub>3</sub>, and d<sub>8</sub>-Cu[N(CN)<sub>2</sub>]Br [9] which are non-metallic at ambient pressure [10, 11]. On the basis of NMR lineshapes, Kanoda [11] has suggested that the magnetic orderings in  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu[N(CN)<sub>2</sub>]Cl and d<sub>8</sub>-Cu[N(CN)<sub>2</sub>]Br are commensurate. The magnetic moment has been estimated to be (0.4–1.0)  $\mu_B$  per dimer. Using uniaxial stress within a layer or changing the anion X, it may be possible to vary the ratio  $J_2/J_1$  and induce a quantum phase transition into a disordered phase or the spiral phase discussed here.

This paper is organized as follows. In section 2, we introduce the model, before presenting the linear spin-wave theory in section 3. In section 4 we present our results. We find that at  $J_2/J_1 = 0.5$  quantum fluctuations are enhanced giving a large but finite correction to the magnetization, suggesting the possibility of having a disordered state. This possibility is also found for  $J_2/J_1 > 4$ : in this region of parameters, the quantum correction to the magnetization is comparable to the classical magnetization and we find that it is much larger than for a set of weakly coupled chains with non-frustrated interchain couplings.

## 2. The model

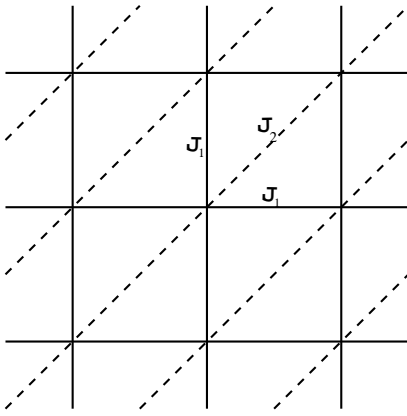
We consider the Hubbard model on the anisotropic triangular lattice with one electron per site. If the Coulomb repulsion between two electrons on the same site is sufficiently large then the ground state is an insulator and a standard strong-coupling expansion for the Hubbard Hamiltonian implies that the spin degrees of freedom are described by a spin- $\frac{1}{2}$  Heisenberg model:

$$H = J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle lm \rangle} \mathbf{S}_l \cdot \mathbf{S}_m. \quad (1)$$

We use the notation  $\langle ij \rangle$  to denote nearest-neighbour bonds and  $\langle lm \rangle$  to denote bonds along the north-east diagonals.  $J_1$  is an antiferromagnetic exchange between nearest neighbours, i.e., along the vertical and horizontal directions, and  $J_2$  is an antiferromagnetic exchange along *one* of the diagonals.  $J_1$  and  $J_2$  are competing interactions leading to magnetic frustration. The model is equivalent to a Heisenberg model on an anisotropic triangular lattice. This lattice is shown in figure 1. Note that the special cases  $J_2 = 0$ ,  $J_1 = J_2$ , and  $J_1 = 0$  correspond to the square lattice, isotropic triangular lattice, and decoupled chains, respectively. The parameter values in reference [2] suggest that  $J_2/J_1 \sim 0.34$ –1 for the  $\kappa$ -(BEDT-TTF)<sub>2</sub>X family and so magnetic frustration will play an important role in these materials.

Insight can be gained by considering various limits of this model:

(i) *The classical limit.* The limit of infinite spin ( $S \rightarrow \infty$ ) corresponds to a classical Heisenberg model. The classical ground state of the system can be computed straightforwardly as a function of the ratio  $J_2/J_1$  assuming that the set of possible spin configurations of the system are correctly described by a spiral form:  $S_i = S u e^{i q \cdot r_i}$ .  $u$  is a vector expressed in terms of an arbitrary orthonormal basis and  $q$  defines the relative orientation of the spins on the lattice. This ground-state configuration was first analysed in [12].



**Figure 1.** The anisotropic triangular lattice, showing the competing interactions  $J_1$  and  $J_2$  that lead to magnetic frustration.

Introducing  $S_i$  in Hamiltonian (1), we get an expression for the classical energy in terms of the spiral vector  $\mathbf{q}$ :

$$\frac{E(q_x, q_y)}{NS^2} = J_1[\cos q_x + \cos q_y] + J_2 \cos(q_x + q_y) \equiv J(\mathbf{q}) \quad (2)$$

where  $N$  is the number of sites on the lattice. For  $J_2 < J_1/2$ , the ground state has Néel order with the associated wave vector  $\mathbf{q} = (\pi, \pi)$ . For  $J_2 > J_1/2$ , the ground state has spiral order with wave vector  $(q, q)$  where  $q = \arccos(-J_1/2J_2)$ .

(ii) *The limiting cases for  $S = \frac{1}{2}$ .* If  $J_2 = 0$  then the model reduces to the Heisenberg model on a square lattice. At zero temperature there will be long-range Néel order with magnetization [13] of  $\langle S_i^z \rangle = 0.3$ . If  $J_1$  is non-zero but small it will introduce a small amount of magnetic frustration which will reduce the magnitude of the magnetization in the Néel state. If  $J_2 = J_1$  then the model reduces to the Heisenberg model on an isotropic triangular lattice. There has been some controversy about the ground state of this model. Anderson [14] originally suggested that the ground state was a ‘spin liquid’ with no long-range magnetic order. However, recent numerical work suggests that there is long-range order but the quantum fluctuations are so large due to magnetic frustration that the magnetic moment may be an order of magnitude smaller than the classical value [15]. If  $J_1$  is non-zero but small we have chains on the diagonals of the lattice that are weakly coupled. The case of only two chains corresponds to the ‘zigzag’ spin chain which is equivalent to a single chain with nearest-neighbour and next-nearest-neighbour exchange,  $J_1$  and  $J_2$ , respectively. This spin chain has been extensively studied and is well understood [16]. In the limit of interest here,  $J_2 \gg J_1$ , there is a gap in the spectrum  $\Delta \sim \exp(-\text{constant} \times J_2/J_1)$  and there are long-range dimer order and incommensurate spin correlations. McKenzie speculated that this ‘spin gap’ is still present in the many-chain limit [2].

### 3. Linear spin-wave theory

Extensive theoretical work has been done with the aim of achieving an understanding of the ground-state properties of the Heisenberg model on the square and isotropic triangular lattices. One standard and simple theory used for calculating the magnetization and energy of the magnetically ordered phases of these systems is linear spin-wave theory (LSW). LSW theory has satisfactorily reproduced the ground-state energy and magnetization of the square [13, 17] and triangular lattices [18, 19]. More sophisticated methods, such as the variational approach of Huse and Elser [20], have corroborated this fact. In the present work, we apply linear

spin-wave theory to the model of interest. Recently, Bhaumik and Bose [21] considered the linear spin-wave theory for the Néel phase ( $J_2/J_1 < 0.5$ ) of the same model. However, they did not evaluate the quantum corrections to the magnetization.

Following Miyake [22] and Singh and Huse [23], it is convenient to rotate the quantum projection axis of the spins at each site along its classical direction. This transformation is carried out by introducing the following rotated spin operators:

$$\begin{aligned} S_i^x &= \hat{S}_i^x \cos(\theta_i) + \hat{S}_i^z \sin(\theta_i) \\ S_i^y &= \hat{S}_i^y \\ S_i^z &= \hat{S}_i^z \cos(\theta_i) - \hat{S}_i^x \sin(\theta_i) \end{aligned} \quad (3)$$

in Hamiltonian (1).

This rotation simplifies the spin-wave treatment with the result that only one type of bosons rather than three is needed to describe the spin operators. After this transformation the Hamiltonian is

$$\begin{aligned} H &= J_1 \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) (\hat{S}_i^x \hat{S}_j^x - \hat{S}_i^z \hat{S}_j^z) + \sin(\theta_i - \theta_j) (\hat{S}_i^z \hat{S}_j^x - \hat{S}_i^x \hat{S}_j^z) + \hat{S}_i^y \hat{S}_j^y \\ &\quad + J_2 \sum_{\langle lm \rangle} \cos(\theta_l - \theta_m) (\hat{S}_l^x \hat{S}_m^x - \hat{S}_l^z \hat{S}_m^z) \\ &\quad + \sin(\theta_l - \theta_m) (\hat{S}_l^z \hat{S}_m^x - \hat{S}_l^x \hat{S}_m^z) + \hat{S}_l^y \hat{S}_m^y - B \sum_i \hat{S}_i^z \end{aligned} \quad (4)$$

where we have introduced an auxiliary magnetic field,  $B$ , in the  $z$ -direction of the rotated basis in order to compute the magnetization.

Using the standard Holstein–Primakoff representation [13] for the spin operators we expand them with respect to  $1/S$  and we take the Fourier transform of the boson operators. The resulting Hamiltonian up to  $O(1/S)$

$$\begin{aligned} H &= J_1 S^2 \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) + J_2 S^2 \sum_{\langle lm \rangle} \cos(\theta_l - \theta_m) - NBS \\ &\quad + S \sum_{\mathbf{k}} [J_1 (A_1(\mathbf{k}) + J_2 A_2(\mathbf{k}) - J_1 C_1) - J_2 C_2 + B/s] a_{\mathbf{k}}^+ a_{\mathbf{k}} \\ &\quad + S \sum_{\mathbf{k}} (J_1 B_1(\mathbf{k}) + J_2 B_2(\mathbf{k})) (a_{\mathbf{k}}^+ a_{-\mathbf{k}}^+ + a_{\mathbf{k}} a_{-\mathbf{k}}) \end{aligned} \quad (5)$$

is not diagonal in the boson operators.  $N$  is the number of lattice sites in the system, and the coefficients  $A$ ,  $B$ , and  $C$  are

$$\begin{aligned} A_1(\mathbf{k}) &= \frac{1}{N} \sum_{\langle ij \rangle} \cos(\mathbf{k} \cdot (\mathbf{r}_j - \mathbf{r}_i)) (\cos(\theta_i - \theta_j) + 1) \\ A_2(\mathbf{k}) &= \frac{1}{N} \sum_{\langle lm \rangle} \cos(\mathbf{k} \cdot (\mathbf{r}_m - \mathbf{r}_l)) (\cos(\theta_l - \theta_m) + 1) \\ B_1(\mathbf{k}) &= \frac{1}{2N} \sum_{\langle ij \rangle} \exp(i\mathbf{k} \cdot (\mathbf{r}_j - \mathbf{r}_i)) (\cos(\theta_i - \theta_j) - 1) \\ B_2(\mathbf{k}) &= \frac{1}{2N} \sum_{\langle lm \rangle} \exp(i\mathbf{k} \cdot (\mathbf{r}_m - \mathbf{r}_l)) (\cos(\theta_l - \theta_m) - 1) \\ C_1 &= \frac{2}{N} \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) \\ C_2 &= \frac{2}{N} \sum_{\langle lm \rangle} \cos(\theta_l - \theta_m) \end{aligned} \quad (6)$$

where again  $\langle ij \rangle$  sums over the nearest neighbours and  $\langle lm \rangle$  over the next-nearest ones.

Hamiltonian (5) explicitly breaks up the SU(2) symmetry of the spins at each site. Only magnetically ordered states can be analysed with this method and the expansion is valid only when the correction due to the zero-point motion of the spins is sufficiently small.

Following standard LSW theory we diagonalize Hamiltonian (5) using a Bogoliubov transformation, assuming, as we have already discussed, a spiral ordering of the spins in the lattice, so that the relative angle between two nearest-neighbour spins is  $q$ , and  $2q$  for two next-nearest neighbours. The diagonal Hamiltonian reads

$$H = NS^2J(q) - NBS - \frac{1}{2}S \sum_{\mathbf{k}} \left\{ \frac{1}{2}(J(\mathbf{k} + \mathbf{q}) + J(\mathbf{k})) + B/S - 2J(q) - \omega(\mathbf{k}, B) \right\} + \sum_{\mathbf{k}} \omega(\mathbf{k}, B) \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}} \quad (7)$$

where the sums run over the first Brillouin zone and  $\alpha_{\mathbf{k}}$  creates boson spin excitations (magnons) with the dispersion relation

$$\omega(\mathbf{k}, B) = 2S[(J(\mathbf{k}) + B/S - J(q))\{\frac{1}{2}(J(\mathbf{k} + \mathbf{q}) + J(\mathbf{k} - \mathbf{q})) + B/S - J(q)\}]^{1/2}. \quad (8)$$

Equation (8) explicitly shows the zero bosonic modes at the momentum wavevectors  $\mathbf{k} = \mathbf{q}$  and  $\mathbf{k} = 0$  for  $B = 0$ .

The ground-state energy of the system,  $E_0$ , is given by equation (7), setting the occupation of the bosons to zero, and, finally, the magnetization of the system is computed as the derivative of the ground-state energy:

$$\langle S_i^z \rangle = -\frac{1}{N} \lim_{B \rightarrow 0} \frac{dE_0}{dB} = S + \frac{1}{2} - S \frac{1}{2N} \sum_{\mathbf{k}} \frac{\frac{1}{2}(J(\mathbf{k} + \mathbf{q}) + J(\mathbf{k} - \mathbf{q})) + J(\mathbf{k}) - 2J(q)}{\omega(\mathbf{k}, B = 0)}. \quad (9)$$

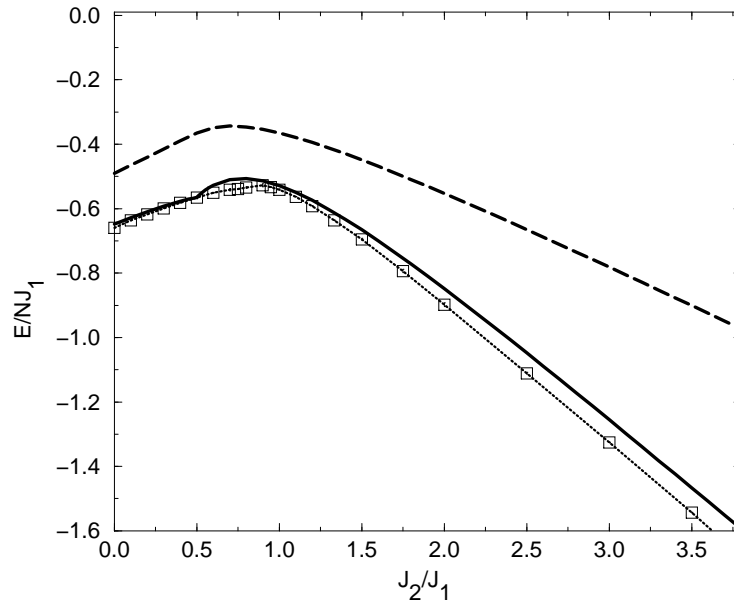
This expression recovers the LSW expression for the magnetization on the square lattice [13],  $\mathbf{q} = (\pi, \pi)$ , and the isotropic triangular lattice [22],  $\mathbf{q} = (2\pi/3, 2\pi/3)$ .

#### 4. Results

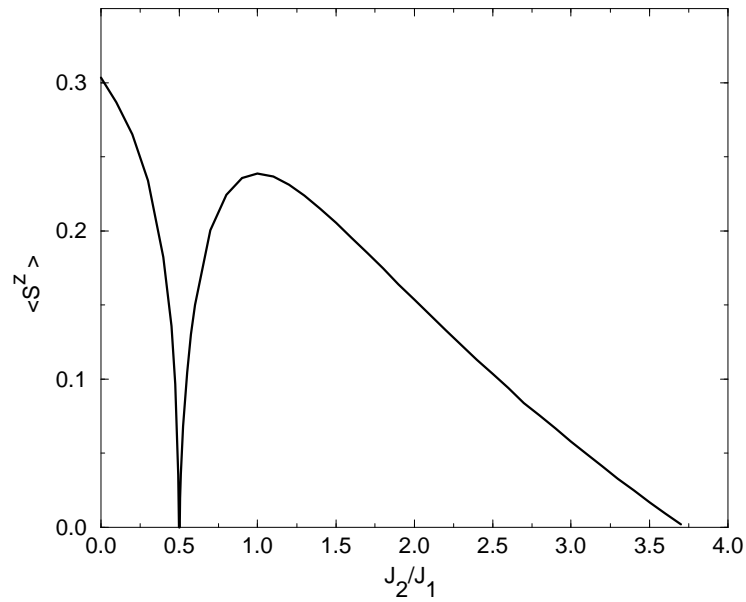
We have numerically evaluated the integrals in equation (7) and (9) to obtain the ground-state energy and magnetization as functions of  $J_2/J_1$ .

##### 4.1. Ground-state energy

In figure 2 we plot the ground-state energy per site as a function of  $J_2/J_1$ . The classical energy is also included in the same figure (dashed curve), showing how quantum fluctuations lower the ground-state energy. The maximum in the total energy is attained at around  $J_2/J_1 \approx 0.7$ , which approximately coincides with the position of the maximum in the classical energy at  $\sqrt{2}/2$ : at this point, geometrical frustration attains its maximum. A cusp is found at the transition from the Néel to the spiral phase at  $J_2/J_1 = 0.5$ , which results, as will be seen later, from the softening of the spin-wave modes at the transition point. We also plot in the same figure results for the ground-state energy obtained from a series expansion calculation [24]. Although LSW theory is a simple approximation for computing ground-state properties of frustrated systems, the energies obtained are in good agreement with the series expansion results.



**Figure 2.** The total ground-state energy (full curve) for  $S = \frac{1}{2}$  and the classical ground-state energy (dashed curve) as functions of the amount of magnetic frustration  $J_2/J_1$ . The squares represent the ground-state energies obtained from a series expansion calculation [24]. The error bars associated with the series expansion energies are, at most, of order  $10^{-3}$ , so they cannot be seen in this plot.



**Figure 3.** The effect of frustration on the magnetization calculated from linear spin-wave theory is shown as a function of  $J_2/J_1$ . The quantum correction to the magnetization diverges as  $J_2/J_1 \rightarrow \infty$ . At the Néel–spiral transition (at  $J_2/J_1 = 0.5$ ) the correction is large due to the softening of the spin-wave modes but finite (see the text). This suggests the possibility of a quantum disordered phase at  $J_2/J_1 \approx 0.5$  and for  $J_2/J_1 > 4$ .

#### 4.2. Magnetization

In figure 3 we show results for the magnetization for  $S = \frac{1}{2}$  as a function of  $J_2/J_1$ . The results are qualitatively similar to those of a recent series expansion study [24]. We have tested the accuracy of the calculation by comparing with known results for the square ( $J_2/J_1 = 0.0$ ) lattice [13] with magnetization  $\langle S_i^z \rangle = 0.303\,39$ , and the triangular lattice [18] ( $J_2/J_1 = 1.0$ ) with  $\langle S_i^z \rangle = 0.238\,68$ . We find a strong dip in the magnetization at  $J_2/J_1 = 0.5$  suggesting the possibility of a disordered phase in its neighbourhood. The nature of the ground state is unclear and will have to be determined by more sophisticated techniques.

It is instructive to mention results reported on other lattices such as the  $J_1$ – $J_2$  square lattice (for which there is frustration along both diagonals), for which extensive work has been carried out. Schulz, Ziman and Poilblanc [25], using exact diagonalization of finite cells, get results for the magnetization in qualitative agreement with LSW theory and a Dyson–Maleev approach performed by Gochev [26] which treats the interaction between the spin waves self-consistently. However, on the isotropic triangular lattice with next-nearest neighbours, Deutscher and Everts [27] have shown that, while exact diagonalization shows a finite jump of the magnetization between the collinear and the  $q = 2\pi/3$  canted phases, LSW theory gives a continuous transition.

From the above discussion we therefore conclude that it is difficult to extract conclusive answers from LSW theory for this frustrated lattice at the transition point: the interaction between the spin waves becomes very large due to the presence of geometrical frustration leading to the possibility of a completely different state to the classical one, and more sophisticated methods are needed to describe this region correctly. A recent series expansion study [24] suggests that the system is Néel ordered for  $J_2/J_1 < 0.7$  and quantum disordered for  $0.7 < J_2/J_1 < 0.9$ .

### 5. Spin-wave velocities and quantum fluctuations

#### 5.1. Near the Néel–spiral transition

Insight into the origin of the increase in the quantum fluctuations near  $J_2/J_1 = 0.5$  can be gained from considering the behaviour of the spin-wave velocities near the zero bosonic modes:  $(0, 0)$  and  $(\pi, \pi)$ . In figure 4, we show the dependences of the magnon frequencies as functions of the wavevector  $k$  along the  $(k, k)$  and  $(k, -k)$  directions in the parameter region  $0 < J_2/J_1 \leq 0.5$  where the  $(\pi, \pi)$  state is classically stable. Except at  $J_2/J_1 = 0.5$ , the magnon excitation vanishes linearly with wavevector, and we define the associated spin-wave velocity along the  $(k, k)$  direction as  $c_+$  and  $c_-$  along the direction perpendicular to it:  $(k, -k)$ .

While near the zero modes,  $(0, 0)$  and  $(\pi, \pi)$ , the spin-wave velocity

$$c_+ \approx 2SJ_1\sqrt{2(1 - 2J_2/J_1)}$$

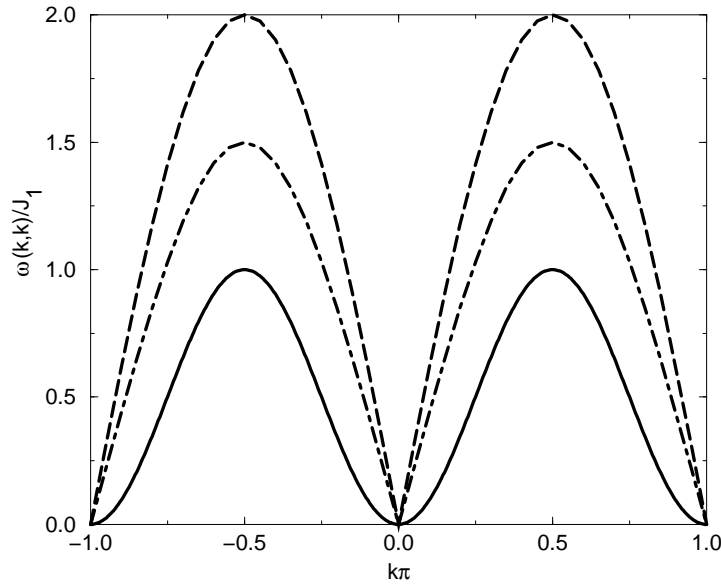
and  $c_-$  does not depend on  $J_2$ :

$$c_- \approx 2\sqrt{2}SJ_1.$$

Hence, the modes soften along the diagonal direction and the velocity  $c_+$  vanishes at the Néel–spiral transition. This can be clearly seen in figure 4.

From the above analysis we can now explain the behaviour of the low-energy, long-wavelength component of the quantum fluctuations as  $J_2/J_1$  approaches 0.5. At this point and near  $(k_x, k_y) \approx (0, 0)$ , the correction to the magnetization due to quantum fluctuations (see





**Figure 4.** The softening of the spin waves near the commensurate–incommensurate transition. The spin-wave dispersion  $\omega(k_x, k_y)$  along the  $(k, k)$  direction is shown for different values of  $J_2/J_1$ . Note that the dispersion near 0 and  $\pi$  becomes quadratic at the transition. As discussed in the text, this leads to an increase in the quantum correction to the magnetization.  $J_2/J_1 = 0$  (dashed curve),  $J_2/J_1 = 0.25$  (dashed-dotted curve), and  $J_2/J_1 = 0.5$  (full curve).

equation (9) for  $S = \frac{1}{2}$  can be approximated by

$$\langle S_i^z \rangle - \frac{1}{2} \approx \frac{1}{2} - \frac{1}{8\pi^2} \int \frac{dk_+ dk_-}{\sqrt{(k_+/2)^4 + k_-^2}} \approx \frac{1}{2} - \frac{k_c}{2\pi^2} (\log(2k_c) - 2 \log(k_c/2) + 2) \quad (10)$$

where the integral is expressed in terms of the diagonal directions,  $k_+ = k_x + k_y$  and  $k_- = k_x - k_y$ , and  $k_c$  is a cut-off in the momentum. As expected, the lowest-order term that appears in the integral along the  $k_+$ -direction is quartic, because the spin-wave velocity vanishes,  $c_+ \rightarrow 0$ . This behaviour of the magnon spectrum, where the softening occurs only along *one* direction, has also been found by Chubukov and Jolicoeur [28] on the isotropic triangular lattice with next-nearest neighbours at the transition from the collinear to the incommensurate phase. They were able to show that the transition point shifts from the LSW theory result when treating the effect of quantum fluctuations in a self-consistent way. This is in agreement with the results obtained using series expansions [24], which give the transition point at  $J_2/J_1 = 0.7$ , instead of at 0.5 as obtained within LSW theory.

As shown in (10) the integral is *finite* but gives a much larger correction than for the square-lattice ( $J_2 = 0$ ) case. We find that taking a cut-off of  $0.1\pi$ , the integral in the correction is two to three times that for the square lattice ( $J_2 = 0$ ), for the same region of integration. Therefore, the reduction in energy of the bosonic modes along the  $k_+$ -direction (see figure 4) is responsible for a finite but large enhancement of the quantum correction to the magnetization.

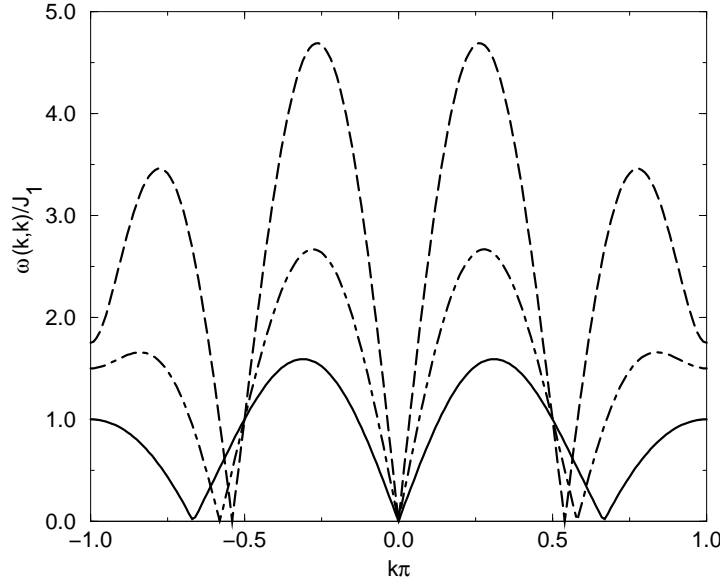
It is interesting to compare this result with the one obtained on the  $J_1$ – $J_2$  square lattice: in this case, the correction due to the quantum fluctuations at  $J_2/J_1 = 0.5$ , can be approximated by

$$\langle S_i^z \rangle - \frac{1}{2} \approx \frac{1}{2} - \frac{1}{8\pi^2} \int \frac{dk_+ dk_-}{\sqrt{(k_+/2)^4 + (k_-/2)^4 - k_+^2 k_-^2 / 8}} = \frac{1}{2} - \frac{1}{(2\pi)^2} \int \frac{dk_x dk_y}{k_x k_y}. \quad (11)$$

The last integral in equation (11) shows a divergence that behaves like the square of a logarithm. Therefore, we find a different qualitative behaviour of the quantum fluctuations for the anisotropic lattice as compared to the  $J_1$ - $J_2$  square lattice.

5.2. Weakly coupled chains ( $J_2 \gg J_1$ )

We have analysed the structure of the magnon excitation spectra in the neighbourhood of the zero bosonic eigen-energies:  $(0, 0)$  and  $(q, q)$ , with  $q \rightarrow \pi/2$  as  $J_2/J_1 \rightarrow \infty$ . We find that the spin-wave velocities are  $c_- \approx \sqrt{2}SJ_1$  and  $c_+ \approx 2\sqrt{2}SJ_2$ . The dispersion relation is plotted in figure 5.



**Figure 5.** The spin-wave dispersion relation along the  $(k, k)$  direction in the spiral phase. As  $J_2/J_1$  increases the limit of weakly coupled chains is approached and the classical ordering wavevector,  $q$ , at which the spin-wave energy vanishes, approaches  $\pi/2$ .  $J_2/J_1 = 1$  (full curve),  $J_2/J_1 = 2$  (dashed curve), and  $J_2/J_1 = 4$  (dashed-dotted curve).

Again, we analyse the low-energy, long-wavelength contributions to the integral appearing in equation (9). Near the zero modes:  $(0, 0)$ , and  $(q, q)$ , it can be approximated by an elliptic integral

$$\int \frac{dk_+ dk_-}{\sqrt{(c_+k_+)^2 + (c_-k_-)^2}} \tag{12}$$

In the limit  $J_2/J_1 \gg 1$ , the ratio of velocities diverges as  $c_+/c_- \approx 2J_2/J_1$ , and the elliptic integral can be approximated by a logarithm. Taking a cut-off of  $\pi/2$ , we get for the magnetization (equation (9))

$$\langle S_i^z \rangle \approx S + 3 \frac{\sqrt{2}}{4\pi} \log(J_1/2J_2). \tag{13}$$

As expected, the quantum fluctuations diverge as  $J_2/J_1 \rightarrow \infty$ . For  $S = \frac{1}{2}$  the critical value that sets  $\langle S_i^z \rangle = 0$  is  $J_2/J_1 \approx 2$ . Although this is just a crude estimate it is roughly consistent with figure 3.

It is interesting to compare this result with the one obtained by Affleck, Gelfand and Singh [29] for the anisotropic  $J_y$ - $J_x$  square lattice where  $J_y$  is a non-frustrating interaction which can be gradually turned off. In the limit  $J_x/J_y \rightarrow 0$ , the integral reduces to

$$\langle S_i^z \rangle \approx S + \frac{1}{2\pi} \log(J_x/J_y) \quad (14)$$

taking a momentum cut-off of  $\pi$ . Note that the factor multiplying the logarithm is smaller in this case than for the anisotropic triangular lattice, and, therefore, the critical value obtained, within spin-wave theory, is an order of magnitude larger:  $J_x/J_y \approx 23.1$ . Numerical work [29] and renormalization group arguments [30,31] on the anisotropic square lattice suggest the existence of long-range Néel order for an infinitesimal coupling between the chains ( $J_x/J_y \ll 1$ ). A similar renormalization group analysis for the appropriate SO(3) non-linear sigma model [32, 33], could be performed to gain some insight into the ground state of our model in this parameter region. Also, numerical calculations using more sophisticated numerical techniques could be performed to find whether long-range order persists for an infinitesimal coupling or not. However, our analysis clearly shows that for comparable interchain coupling the quantum fluctuations are much larger than for the case where the interchain coupling is non-frustrated.

## 6. Conclusions

We have presented a linear spin-wave analysis of the antiferromagnetic Heisenberg model on an anisotropic triangular lattice. The Heisenberg model on this lattice should be the relevant model for describing the insulating properties of certain layered organic superconductors. The correction to both the energy and magnetization due to quantum fluctuations is computed by means of linear spin-wave theory for different values of  $J_2/J_1$ . The results obtained from LSW theory suggest the possibility of finding a disordered state near  $J_2/J_1 \approx 0.5$ . This possibility also exists for values of  $J_2/J_1 > 4$ , where the system resembles a set of chains weakly coupled by a frustrated interaction. In this region of parameters we find that quantum fluctuations are larger than in the case where the chains are weakly coupled by a non-frustrated interaction. Further work using other numerical approaches should be carried out to analyse in more detail the results presented here.

## Acknowledgments

We thank J Oitmaa and R R P Singh for helpful discussions. We thank W Zheng for providing us with the series expansion data. Computational work in support of this research was partly performed on Cray computers at Brown University's Theoretical Physics Computing Facility. J Merino was supported in part by a NATO postdoctoral fellowship and the Australian Research Council. This work was supported in part by the National Science Foundation under contracts NSF DMR-9357613 (JBM and CHC).

*Note added.* When we were completing this paper we became aware that Trumper [34] had obtained some of the results presented here.

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